

Exercise: Chi-Square Test

Human Computer Interaction

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Portale della Didattica: A/B Testing

Will the "Community" link lead to significantly more students using the functionality versus the original "Forum" link?



Experimental Design: online A/B test, **randomly** show each student one version of the page. We are measuring the "engagement rate", i.e., how many students open that functionality

Portale della Didattica: Collected Data

	Button on the page	
	Community	Forum
Students use the functionality	30	20
Students didn't use the functionality	70	100
Total visitors:	100	120

Portale della Didattica: Collected Data

	Button on the page	
	Community	Forum
Students use the functionality	30 (30%)	20 (17%)
Students didn't use the functionality	70	100

Total visitors:

100

120

30% is higher than 17%, so we are done!

Right? "Community" is clearly better!

Portale della Didattica: Collected Data

- Not so fast!
- What if this happened just by chance since we had quite few visitors?
- Let's use the Chi-square test to understand whether this difference is significant:
 - *null hypothesis*: _____
 - *alternative hypothesis*: _____

Portale della Didattica: Collected Data

- Not so fast!
- What if this happened just by chance since we had quite few visitors?
- Let's use the Chi-square test to understand whether this difference is significant:
 - *null hypothesis*: the "Community" link will lead to no significant change in the number of students using the functionality versus the original "Forum" link
 - *alternative hypothesis*: the "Community" link will lead to significant more students using the functionality versus the original "Forum" link

Portale della Didattica: Collected Data

	Button on the page		Totals
	Community	Forum	
Students use the functionality	30	20	50
Students didn't use the functionality	70	100	170
Total visitors:	100	120	220

What do we expect to have if the null hypothesis is true?

Portale della Didattica: Collected Data

	Button on the page		Totals
	Community	Forum	
Students use the functionality	30	20	50
Students didn't use the functionality	70	100	170
Total visitors:	100	120	220

$$\frac{(row_{total} * col_{total})}{grand_{total}}$$
 Expected

	Button on the page		Totals
	Community	Forum	
Students use the functionality			50
Students didn't use the functionality			170

An arrow points from the formula to the empty cell in the first row and first column of the 'Expected' table.

Portale della Didattica: Collected Data

	Button on the page		Totals
	Community	Forum	
Students use the functionality	30	20	50
Students didn't use the functionality	70	100	170
Total visitors:	100	120	220

Expected

	Button on the page		Totals
	Community	Forum	
Students use the functionality	$(50/220)*100 = 22.7$	$(50/220)*120 = 27.3$	50
Students didn't use the functionality	$(170/220)*100 = 77.3$	$(170/220)*120 = 92.7$	170

Portale della Didattica: Collected Data

	Button on the page		Totals
	Community	Forum	
Students use the functionality	30	20	50
Students didn't use the functionality	70	100	170
Total visitors:	100	120	220

Expected

	Button on the page		Totals
	Community	Forum	
Students use the functionality	22.7	27.3	50
Students didn't use the functionality	77.3	92.7	170

Chi-Square Test: Process

1. Calculate

- $\chi^2 = \sum_{i=1}^n \frac{(O_i - E_i)^2}{E_i}$

- where O_i is the i -th observation and E_i is the expected (theoretical) count of type i

- $$\chi^2 = \frac{(30-22.7)^2}{22.7} + \frac{(20-27.3)^2}{27.3} + \frac{(70-77.3)^2}{77.3} + \frac{(100-92.7)^2}{92.7} =$$
$$2.34 + 1.95 + 0.69 + 0.57 = 5.55$$

Chi-Square Test: Process

2. Determine the degrees of freedom, df , of that statistic:
 - Test of independence, $df = (Rows - 1) * (Cols - 1)$
 - $df = (2 - 1) * (2 - 1) = 1$

Chi-Square Test: Process

- Look for the level of confidence (p-value) related to the χ^2 result (5.55) and df (1) in a Probability Table:

df	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	---	---	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750

from <https://people.richland.edu/james/lecture/m170/tbl-chi.html>

- Coin example:
 - first row, $p \approx 0.025$

Chi-Square Test: Process

4. **Sustain** or **reject** the null hypothesis

- we usually reject the null hypothesis at $p < 0.05$ or $p < 0.01$
 - i.e., we are confident that 95% or 99% of the time the test result correctly applies to the entire population
- In our case:
- we can reject the null hypothesis (if we choose $p < 0.05$)!
 - so, we can say that the (imaginary) "Community" link leads to significant more students using the functionality



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